

Homework 14

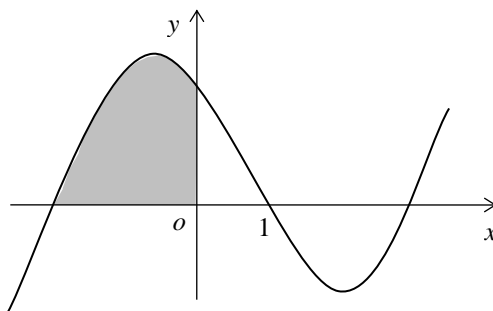
Due Tuesday 17th of April

There are 139 marks so this should take 2 to 3 hours.

1. Verify that $x+3$ is a factor of $2x^3 + 3x^2 - 11x - 6$ and hence factorise the expression completely.
2. Find the equation of the tangent to the curve $y = 8x + \frac{1}{x}$ at the point where $x = \frac{1}{2}$.
3. Find the equation of the tangent to the circle $x^2 + y^2 + 2x - 4y - 5 = 0$ at the point $(2,3)$ on the circle.
4. Show that the line $y = 2x - 8$ is a tangent to the circle with equation $x^2 + y^2 - 4x - 2y = 0$, and find the coordinates of the point of contact.
5. Given that $v = \sqrt{(2x+1)}$, find
 - a. $\frac{dv}{dx}$
 - b. x given that $\frac{dv}{dx} = 5$.
6. Given that $f(x) = \sin 4x - 2\cos^2 x$ evaluate $f'(\frac{\pi}{4})$.
7. Triangle ABC has vertices $(-2,5)$, $(6,1)$ and $(2,-7)$ respectively.
 - a. Find the equation of the perpendicular bisector of BC.
 - b. Show that this perpendicular passes through the mid-point of side AC.
 - c. The perpendicular produced meets the line through A with gradient $\frac{3}{4}$ at T, find the coordinates of T.
8. A and B are the points with coordinates $(3,0,1)$ and $(4,0,2)$ respectively.
 - a. Given that $\vec{BP} = k\vec{AB}$, establish the coordinates of P in terms of k.
 - b. Given now that OP is perpendicular to AB, find the value of k.
9. Given that $\log_2(x+1) - \log_2 x = \log_2 8$, find the value of x.
10. For what values of k does the quadratic equation $kx^2 + 2x + (6k-1) = 0$ have real roots?
11. A, B and C are the points $(\frac{7}{2}, -1, -2)$, $(2, 1, -3)$ and $(\frac{13}{2}, -5, 0)$.

Show that A, B and C are collinear and state the value of the ratio $\vec{BA} : \vec{AC}$.

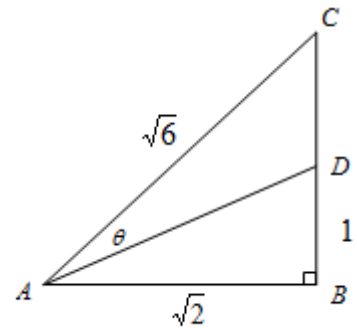
12. The diagram below shows a sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 2x^2 - 5x + 6$.



- a. Show that $x-1$ is a factor of this function and hence solve $f(x) = 0$.
 - b. Calculate the shaded area.
13. Solve the equation $2\cos 2x^\circ = \sin x^\circ - 1$, for $0 \leq x < 360$.

14. In the diagram opposite, show that the exact value of

$$\cos \theta = \frac{4}{3\sqrt{2}}.$$



15. A competitor in a long distance yacht race loses his mast in a storm. Now drifting helplessly he decides to ration his food supply. The only food on board is 16 kg of salted fish. He decides to eat 10% of his remaining food each day and is confident he can catch one fish each night (on his solitary hook) weighing an average of 0.6 kg.

- How long is it before his supply drops below 12 kg?
- Will he ever run out of food?

16. The functions $f(x) = 4 - x^2$ and $g(x) = 2k - x$, where k is a constant, are defined on the set of *positive real numbers*.

- Show that $f(g(x)) + 2[g(f(x)) + 4k]$ can be written as $x^2 + 4kx - 4(k^2 - 3k + 1)$.
- Hence find the values of k such that the equation $f(g(x)) + 2[g(f(x)) + 4k] = 0$ has equal roots.

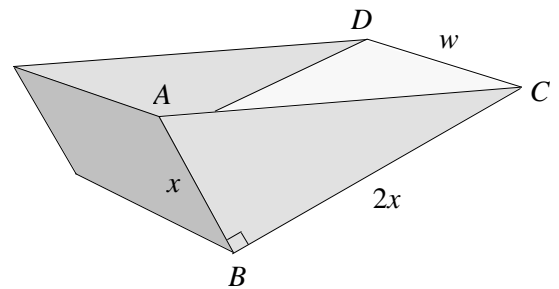
17. Find (a) $\int \frac{x^3 - \sqrt{x}}{x^2} dx$ (b) $\frac{dy}{dx}$ when $y = \left(x^2 + \frac{1}{x}\right)^2$.

18. Tin sheeting is bent and sealed to form a feeding trough in the shape of the prism opposite. Angle ABC is a right-angle. The total amount of tin plate used is $6\frac{1}{2}$ square metres.

$AB = x$, $BC = 2x$ and $CD = w$.

- Show that the surface area, A , in terms of x and w can be written as $A = 2x^2 + 3xw$.

- Hence show that $w = \frac{13}{6x} - \frac{2x}{3}$.



- If the volume of the trough is given as $V \text{ m}^3$, show that $V(x) = \frac{13x}{6} - \frac{2x^3}{3}$.

- Hence find the values of x and w for maximum volume. **Give your answers correct to 2 decimal places.**

19. The points A, B and C have coordinates (2, 1, 1), (-1, 0, 2) and (3, -4, -1) respectively. Calculate the size of angle ABC.

20. A function is defined as $f(x) = x^3 - 3x + 2$, for $x \in \mathbb{R}$.

- Show that $x = -2$ is a root of the equation $x^3 - 3x + 2 = 0$ and hence find the other real root.
- Find the coordinates of the stationary points of the graph $y = f(x)$ and determine their natures.
- Sketch the graph of $y = f(x)$ marking clearly all relevant points.

21. Express $2x^2 - 12x + 1$ in the form $a(x+b)^2 + c$ and write down the values of a , b and c .

22. Express $\cos \theta + \sqrt{3} \sin \theta$ in the form $W \cos(\theta - \alpha)$, where W is a constant and $0 \leq \alpha < 2\pi$.

23. A radioactive substance decays according to the formula $M_t = M_o \cdot 8^{-0.3t}$, where M_o is the initial mass of the substance, M_t is the mass remaining after t years. Calculate, to the nearest day, how long a sample would take to half its original mass.